Topological Materials and Applications in Computing

Abstract

The field of topological materials sits at the intersection of math and physics and studies exotic materials with novel properties, like a substance with an insulating interior but an indestructible conducting surface. Topological materials present possibilities in every field where cutting-edge materials are in need, most notably computing. Yet outside of research, the field remains largely in obscurity, which can be partially attributed to the field's deep theoretical and mathematical underpinnings. In this paper I introduce the necessary theoretical background to give a cursory overview of the field. I discuss in specific the subfield of topological insulators and give an example of ongoing research. Finally, I discuss computing applications in which topological materials may hold an important role.

1 Introduction to Topological Materials

The field of computing is at a crossroads. The conventional scaling techniques that powered the exponential growth of processing performance for the past half century are reaching their theoretical limits, with prominent leaders in computing—such as Jensen Huang, the CEO of Nvidia—declaring that Moores law is already dead [1]. Meanwhile, the demand for compute grows greater and greater, with industries such as information technology, financial technology, scientific computing, and artificial intelligence pushing for evermore powerful machines capable of consuming larger datasets and hosting more complex algorithms.

Nearly all of todays processors are manufactured using the CMOS process. The logic family, introduced in the 60s, consumes no power in steady state and is highly resistant to noise. These favorable characteristics caused it to quickly become the dominant process to create logic circuits. While initial increases in transistor densities were facilitated by shrinking feature sizes, more drastic changes were needed as miniaturization of the planar transistor became impossible. Todays processors use the FinFET transistor morphology and tomorrows may use the GAAFET (Gate-All-Around) morphology. Both reconfigure the layout of the transistor to increase transistor density and introduce more ways to shrink features.

However, as manufacturing processes become more and more complex, researchers and industry leaders are looking away from CMOS for the next generations computing technology [2]. Some promising architectures redefine the way the transistor is switched. Tunnel field-effect transistors (TFETs) exploit quantum tunneling to switch current, while spin transistors use the two spin states of electrons (spin up and spin down) to store and manipulate information. Other technologies move away from the electron as the computing medium. Photonic computing uses light to perform operations, offering higher energy efficiency than conventional computers. Quantum computing operates on qubits instead of bits, allowing it to perform certain calculations at speeds exponentially faster than conventional computers.

It is evident that as computing technologies becomes more exotic, so do the materials required to construct devices. For example, preserving the entangled states of qubits is notoriously difficult, requiring todays quantum computers to operate at temperatures near absolute zero. This requires bulky cooling equipment that inflates quantum computers to the size of rooms. For quantum computing to become mainstream, that requirement needs to be eliminated, requiring exotic materials that could host qubits without near-zero temperatures.

The relatively obscure field of topological materials may hold solutions for many of these future technologies. Existing at the intersection of mathematics, theoretical physics, and semiconductors, the field studies exotic substances with perplexing properties, such as insulating materials with conducting surfaces that are impossible to puncture.

In this paper, I will introduce the mathematical and theoretical origins of topological materials and demonstrate how it helps us characterize certain material systems. Of specific research interest is the newer subfield of topological insulators, which I will examine in detail. As a specific example, I will highlight the work of Professor Simmonds, whose research into topological insulators is ongoing right here at Tufts. Finally, I will discuss applications of the field in various upcoming technologies.

2 Theoretical Background

The field of topological materials is extremely math and theory heavy—the mathematical groundwork for the field was established decades before the first topological materials were experimentally verified. In this section I will introduce the relevant concepts to the extent required for a cursory understanding of the field. Of interest is topology and quantum mechanics, specifically the second quantization.

2.1 Topology

Topology is often called rubber-sheet geometry [3]. Like geometry, it is concerned with the properties of shapes and objects in space [4]. Unlike geometry, however, exact lengths and positions arent relevant—objects can be

squeezed, stretched, and pushed around while still considered the same: its as if the objects themselves were made out of rubber. These smooth operations are called homeomorphisms, from Greek meaning "similar shape." The objects they act on are called topological spaces because we can imagine their surface as a space on which points can be located.

When a topological space undergoes a homeomorphism, certain properties are preserved. For example, when we distort a donut into a mug, the number of holes is preserved: the hole of the donut becomes the handle of the mug. A homeomorphism cannot change these properties; hence, they are called topological invariants. A ball and a mug would have different topological invariants because it is impossible to turn one into the other without poking or removing a hole—actions that are not continuous. Indeed, a ball has zero holes while a mug has one, showing that the number of holes would function as a topological invariant. This specific topological invariant is called the genus of a surface.

However, to really understand a topological invariant, we would want a way to calculate the invariant of a particular topological space without relying on ambiguous definitions like the number of holes. (Ask a couple of people to count the holes in a pair of pants and youre bound to get different answers.) Hence, when examining a new class of topological objects, finding a method to calculate the invariant is usually of primary interest. Fortunately, for surfaces, the Gauss-Bonnet theorem proves that the following integral of the Gaussian curvature, K, is equal to a topological invariant known as the Euler characteristic [5].

$$\chi = \frac{1}{2\pi}\int_S K dA$$

The genus can then be calculated from the Euler characteristic with the following relation: $\chi = 2 - 2g$.

Armed with the topological invariant, how do we proceed? Well, using the invariant, we can partition a set of objects into sectors: objects with the same invariant will go into the same sector and those with different invariants will go into different sectors. We can then label each sector with an element from an overarching set. This overarching set gives deep insight into the rules we are using and the objects we are classifying. For example, the set we would use to label our surfaces would be $\mathbb{Z}$, the integers. It suggests that there are a countably infinite number of shapes that cannot be transformed into each other.

2.2 Second Quantization

In the real world, we do not have the luxury of continuous spaces and infinitely dividable surfaces. Instead, as we observe smaller and smaller scales, we encounter an entirely different regime governed by rules alien to our understanding of the macroscopic world. This is, of course, the study of quantum mechanics.

Recall the derivation of the Schrodinger equation by applying de Broglies particle-wave equivalencies for energy and momentum to the classical energy and momentum relation [6]. We treat the result as a dispersion relationship and get the following equation of the wavefunction.

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V(x,t)\Psi$$

Typically, this is a rather cumbersome way of working with the Schrodinger equation. We would like to combine the kinetic energy and potential energy terms on the right into a single operation. This is done using the Hamiltonian operator (as denoted by $\hat{H}$), which encompasses all energies of the system.

$$i\hbar\frac{\partial\Psi}{\partial t} = \hat{H}\Psi,\ \hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x,t)$$

However, we are still far from modeling any real-world system—the equation only describes a single particle. As it turns out, if we want to model multiple particles, we need more than just a single one-dimensional wavefunction for each particle [7]. Rather, we need a n-dimensional wavefunction, where n is the number of particles—the complexity of the wavefunction increases exponentially as we add more particles. Some quantum mechanical phenomena allude to this complexity: for instance, quantum entanglement describes a unique relationship between two or more particles, so it would be impossible to exhibit such a phenomenon when manipulating each wavefunction separately.

Adding to the complexity of the many-body Schrodinger equation is that particles are indistinguishable. Suppose I have a particle in one position and a particle in another. The particles are identical: same type and same properties. If you were to have looked away, you would not be able to tell whether I had swapped them or not. This symmetry must be encoded in the Schrodinger equation. Specifically, for our two-particle system, the following relation must be true.

$$\psi(r_1, r_2) = \pm\psi(r_2, r_1)$$

The coefficient depends on the type of particle exchanged. For fermions, such as protons, neutrons, and electrons, the coefficient is negative, and for bosons, such as photons, the coefficient is positive.

Clearly all these symmetries are making our many-body Schrodinger equation more and more difficult to solve. If only there was a representation of our system in which these symmetries were naturally encoded...

Enter the second quantization. Instead of denoting our system with the state each particle is in, we denote the number of particles in each state, called the occupation numbers [8]. For example, $|0,1,1,0\rangle$ would represent a four-state system with two particles in the second and third state. If we were to swap the two particles, the representation would be identical, reflecting the indistinguishability of the two particles. Conveniently, this representation also allows us to create and remove particles

and encode other properties of our particles, such as the Pauli exclusion principle.

Specifically, the creation operator $\hat{c}^\dagger$ and annihilation operator $\hat{c}$ are used to add and remove particles. The operator acts upon a state, resulting in a state with one more or fewer particle. To represent an operation that is impossible, such as removing a particle from an empty state, the result should be 0, indicating that no such state exists.

Lets model a simple system using the second quantization. Suppose we have a one-dimensional string of sites on which electrons can reside. We assume that the electrons are spinless, so only one electron can occupy each site (a state). The creation and annihilation operators would then satisfy:

$$\hat{c}^\dagger|0\rangle = |1\rangle,\ \hat{c}|0\rangle = 0,$$
$$\hat{c}^\dagger|1\rangle = 0,\ \hat{c}|1\rangle = |0\rangle,$$

We would also like to describe how particles can move between states. We want something similar to the Hamiltonian operator in the first quantization: an operator that acts on the current state and describes how the system will change. In the second quantization, this is also called the Hamiltonian. However, it operates on occupation numbers rather than the wavefunction using creation and annihilation operators rather than derivatives.

In our system, the electrons move between adjacent states with the hopping amplitude t. We also want to maintain a roughly constant density of electrons μ, which can be done using the density operator $\hat{n}$. The Hamiltonian is as follows:

$$H = \sum_i^L t\hat{c}_i^\dagger\hat{c}_{i+1} + t\hat{c}_{i+1}^\dagger\hat{c}_i + \mu\hat{n}_i$$

Notice that the creation and deletion operators are applied in series. The first operator creates an electron at the adjacent site and the second removes it at the current site, effectively making the electron hop. This action is characterized by the hopping amplitude coefficient t.

From here, we can derive the dispersion relation for the system. Well define the creation and annihilation operators as functions of momentum. Recall that momentum is the Fourier transform of position and that a single position is represented by infinitely many momenta. In this case, momentum, k, is restricted by the spacing between sites: $k = 2\pi n/L$, where n is any integer and L is the length of the system.

$$\hat{c}_i^\dagger = \frac{1}{\sqrt{L}}\sum_k e^{ikx_i}\hat{c}_k^\dagger$$

$$\hat{c}_i = \frac{1}{\sqrt{L}}\sum_k e^{-ikx_i}\hat{c}_k$$

With substitution we get the following, defining $a = x_i - x_{i-1}$ as the spacing between sites.

$$H = \frac{1}{\sqrt{L}}\sum_{ikk'} t\left(e^{-i(k-k')}e^{-ika} + e^{-i(k'-k)}e^{ik'a} + \mu\right)\hat{c}_k^\dagger\hat{c}$$

After a series of simplification steps, we get:

$$H = \sum_k E(k)\hat{c}_k^\dagger\hat{c},\ E(k) = 2t\cos(ka) + \mu$$

Notice that we've expressed the Hamiltonian in a way where particles dont interact with each other; this is known as "diagonalizing the Hamiltonian." Additionally, weve expressed the energy of each state as a function of momentum; this is the dispersion relation. The dispersion relation of this system is plotted in Figure 1.

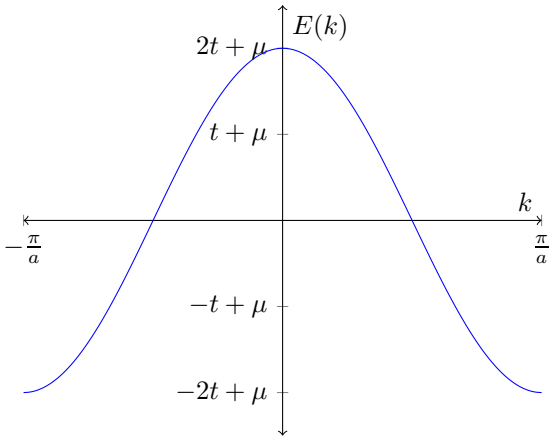

Figure 1. Dispersion relation derived from diagonalizing the Hamiltonian. All states below μ (the y axis) are filled.

3 Topological Materials

The field of topological materials is principally concerned with how deformations of $\hat{H}$, the Hamiltonian, affects $E(k)$, the dispersion relation. Specifically, the field is interested in critical deformations of H that result in the dispersion relation changing its fundamental character, e.g. a bandgap appearing or disappearing. To do so, the set of potential dispersion relations is characterized using topology, giving the field its name.

3.1 First Generation Topological Materials

The application of topology in physics was first proposed to explain the quantum hall effect (QHE) [9]. The quantum hall effect is the observation that the hall resistance of materials in low temperatures and high magnetic fields take on quantized resistance values. Specifically:

$$\sigma_{xy} = n\frac{e^2}{h}$$

Thouless, Kohmoto, Nightingale, and den Nijs (TKNN) showed that the discrete values taken on by resistance corresponded to values of certain topological invariant that classified the materials Hamiltonian. The specific proof of the invariant is beyond the scope of this discussion, but it involves showing that the Hamiltonian of the material and a torus share the same topological invariant, the Chern number. The Chern number can be calculated by integrating $\mathscr{F}_m$, the Berry flux (the flux associated with a potential related to the wavefunctions phase), over the Brillouin zone (the equivalent of the lattice's unit cell in momentum space):

$$n = \sum_m \frac{1}{2\pi} \int d^2\mathbf{k} \mathscr{F}_m$$

However, a less mathematically intense and more intuitive explanation of the QHE is possible. Imagine electrons traveling in a finite two-dimensional surface with a strong applied magnetic field, such as the electrons in our hall bar [10]. We know that moving charges in a magnetic field curve to form orbits, as shown in Figure 2. However, at the edges of the material, the electrons cant complete their orbit, instead bouncing back within the surface as they reach the edges; these are called skipping orbits.

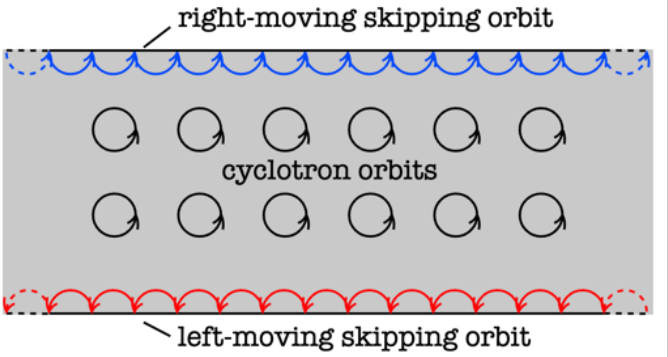

Figure 2. Electrons traveling through a surface with a perpendicular applied magnetic field take on circular orbits. However, electrons at the edges are forced to bounce back, forming skipping orbits.

The skipping orbits are unique because electrons are only able to travel in one direction (relative to the edge and applied magnetic field): they cannot backscatter. As a result, skipping orbits have the highest conductance possible, which is determined by quantum limits to be $G_0 = e^2/h$. The quantized hall effect is then simply the conductivity of a single skipping orbit, e^2/h, multiplied by the number of skipping orbits, n.

3.2 The Tenfold Way

After the discovery of the TKNN invariant, it became of interest to topologically classify more material systems. As it turns out, it only takes ten discrete classes to do so [8]. The class of a particular material system depends on the presence or absence of certain symmetries in it.

The three symmetries of interest are time reversal symmetry, denoted by the operator $\hat{T}$, particle-hole symmetry, denoted by the operator $\hat{C}$, and chiral symmetry, denoted by the operator $\hat{S}$. The material is classified by determining whether the symmetry exists in the system and how it acts if it does.

The time reversal symmetry operator acts by reversing time: On action, position is preserved but momentum (being the derivative with respect to time) is negated. Whether a system is time-reversal symmetric depends on whether the Hamiltonian commutes with the operator, i.e. . If it does, we can further characterize the system based on how it is affected after applying the operator twice ($\hat{T}^2$). Since the operator cant affect the energy of the system, it can only affect the phase of the wavefunction. This results in two values of ($\hat{T}^2$): 1 and -1. In total, there are 3 possibilities for time-reversal: absent, $\hat{T}^2 = 1$, and $\hat{T}^2 = -1$.

The particle-hole symmetry operator turns particle creation into hole creation. Like time reversal symmetry, the symmetry can also take three values: absence, where the operator doesnt commute with the Hamiltonian, $\hat{C}^2 = 1$, where it does and applying it twice preserves phase, and $\hat{C}^2 = -1$, where applying it twice changes the phase by π. Combined with time reversal symmetry, this results in 9 possible classes total.

Chiral symmetry is different: it can be seen as the composition of the other two operators, $\hat{S} = \hat{C} \cdot \hat{T}$. As a result, it is uniquely determined when the other two symmetries are present, preventing it from creating unique classes. However, when both time reversal symmetry and particle-hole symmetry are absent, chiral symmetry can take on two values: absence and presence. Unlike the other two operators, applying it twice always preserves phase. As a result, it only creates one more class.

The particular topological invariant for each of these classes is outside the scope of this discussion, but Table 1 shows the labeling set of the invariant under each class and number of dimensions [11]. This table is called the periodic table of topological invariants.

We see that when the invariant exists, it takes on two values: $\mathbb{Z}$, the natural numbers, or $\mathbb{Z}_2$, the set $\{0, 1\}$. The $\mathbb{Z}_2$ invariant is of particular interest because it characterizes the second generation of topological materials, topological insulators.

3.3 Topological Insulators

Topological insulators (TIs) are a more recently studied subset of topological materials with the following properties: they have time reversal symmetry, specifically $\hat{T}^2 = -1$, and they are characterized by the $\mathbb{Z}_2$ invariant [12]. Specifically, when the $\mathbb{Z}_2$ invariant is 0, the material is said to be topologically trivial, and when the invariant is 1, the material is nontrivial.

The physical interpretation of the invariant can be considered under the spin-orbit coupling (SOC) effect. The

		$\hat{C}$	$\hat{T}$	$\hat{S}$	$d=1$	$d=2$	$d=3$
standard (Wigner-Dyson)	A (unitary)	0	0	0	-	$\mathbb{Z}$	-
	AI (orthogonal)	+1	0	0	-	-	-
	AII (symplectic)	-1	0	0	-	$\mathbb{Z}_2$	$\mathbb{Z}_2$
chiral (sublattice)	AIII (chiral unity)	0	0	1	$\mathbb{Z}$	-	$\mathbb{Z}$
	BDI (chiral orthogonal)	+1	+1	1	$\mathbb{Z}$	-	-
	CII (chiral symplectic)	-1	-1	1	$\mathbb{Z}$	-	$\mathbb{Z}_2$
BdG	D	0	+1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	-
	C	0	-1	0	-	$\mathbb{Z}$	-
	DIII	-1	+1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
	CI	+1	-1	1	-	-	$\mathbb{Z}$

Table 1. The "periodic table of topological invariants" classifies topological materials based on the presence and absence of symmetries. The first two columns classify the material type (standard, chiral, or Bogoliubov-de Gennes) and symmetry classes within. The next three columns specify the symmetries present within the class (time reversal, particle-hole, and chiral). +1 or -1 indicates the phase change upon double application of a symmetry while 0 denotes the lack of a symmetry. Note that chiral symmetry doesn't cause phase change and can only take the value of 1. The last three columns list the labeling sets for the topological invariant (if it exists) as a function of the symmetry class and number of dimentions.

spin-orbit coupling effect originates from the magnetic interaction of an electrons spin and the magnetic dipole generated by a moving charge (i.e. the electron). As spin can only take on two values (called up and down), the effect adjusts the energy of electron orbitals higher or lower by small amounts depending on whether the orbitals spin is oriented with or against the magnetic field. This results in the fine structure of spectral lines in atoms; the observation that a single emission or absorption line corresponding to the energy difference between two electron orbitals is actually composed of two or more neighboring but distinct spectral lines.

However, in certain materials, the SOC effect is strong enough to cause adjacent orbitals to cross. For example, Figure 3 shows how in the well-known topological insulator Bi_2Se_3, the bismuth orbital and the selenium orbital swap energies when SOC is considered [13]. Figure 4 shows the dispersion relationship as SOC becomes more intense [12].

Alone, the SOC effect would not be noteworthy: though consisting of different orbitals, the conduction and valence bands still exist with a band gap in between, causing the material to act like an insulator. However, at the intersection of two materials where SOC only causes inversion in one, something interesting happens: the orbitals must cross, momentarily causing the bandgap to close. This is the critical point in Figure 4, and because the band gap is now closed, the material conducts.

The $\mathbb{Z}_2$ invariant is nontrivial when the orbitals cross due to SOC and is trivial when they dont. The vacuum state, however, is trivial; therefore, the surface of any nontrivial material (i.e. topological insulators) must have a conducting surface. This results in the topologically protected conducting surface of topological insulators: no matter how the material is distorted, the surface of the material will stay conducting while the interior stays insulating.

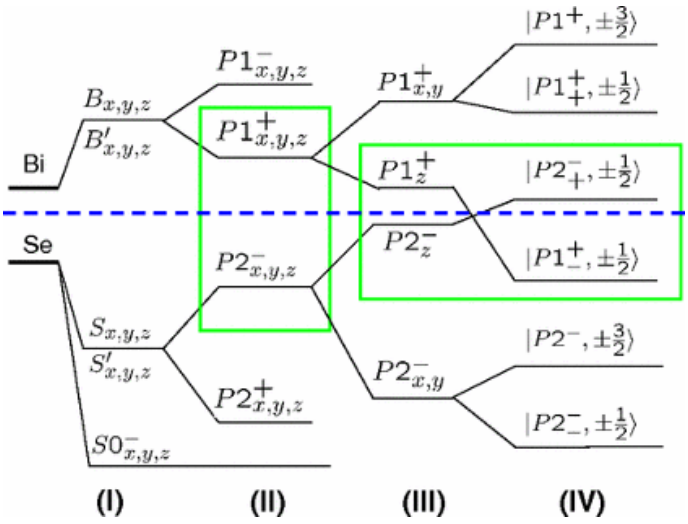

Figure 3. Orbital from bismuth crosses orbital from selenium after consideration of spin-orbit coupling (IV).

The surfaces of topological insulators exhibit another unique phenomenon: spin-momentum locking. The effect describes how the spin of an electron traveling across the materials surface is always locked perpendicular to its direction of motion. The effect results from certain features on the TIs 2D dispersion relation that resemble two cones stacked against each other with touching points, called Dirac cones. Figure 5 shows the Dirac cone with electron spins shown as arrows. As according to SOC, electrons with a spin oriented one way relative to momentum (blue) take lower energies than electrons oriented the other way. Since the Fermi level exists at the intersection of the cones, electrons end up only taking the blue orientation, locking spin respective to momentum.

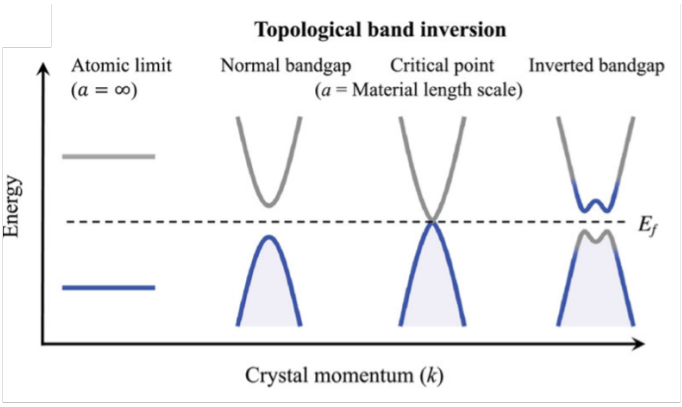

Figure 4. Dispersion relation as SOC becomes more intense. At the critical point, the two orbital energies meet, removing the band gap. As SOC becomes even more intense, orbitals originally in the valence band (blue) form part of the conduction band (grey) and vice versa.

An interesting side effect of spin-momentum locking is backscattering protection. For an electron to reverse its momentum (by scattering off the crystal lattice), the electron must simultaneously reverse its spin. This is, however, extremely unlikely, protecting electrons from backscattering and creating behavior like superconductivity.

3.4 An Example of Ongoing Research

Research into topological materials continues to be very intense. A couple of potential research directions include: studying other unique states like saddle-like states, helicoid states, Seifert states, and linked-node states on TI surfaces; constructing TI-based magnets with unique electromagnetic properties like the layer Hall effect; advancing theory to find more unique topological materials; developing TIs capable of hosting exotic particles like Majorana fermions; and much more [12].

I would like to highlight one ongoing research effort at Tufts University, of which I am lucky to be a part of. The Simmonds lab is working with S. Schmid et al. to develop topological insulators based on InAs/GaSb quantum wells [14].

The following is a brief description of our work. The InAs/GaSb material system is unique in that it has a broken-gap Type-III band alignment: the top of the GaSb valence band is above the bottom of the InAs conduction band. When the two materials are layered, this results in GaSb valence band holes positioned beside InAs conduction band electrons. Typically, electrons from GaSb would then simply tunnel to recombine with holes from InAs, but because the two layers are thin enough to form quantum wells, electron and hole energies are quantized and take different energies.

By adjusting the widths of the two quantum walls, S. Schmid et al. was able to adjust hole energies above electron energies similarly to the configuration within topological insulators, and as a result, exhibit nontrivial behavior like the creation of Van Hove singularities, a unqiue feature in the materials density of states.

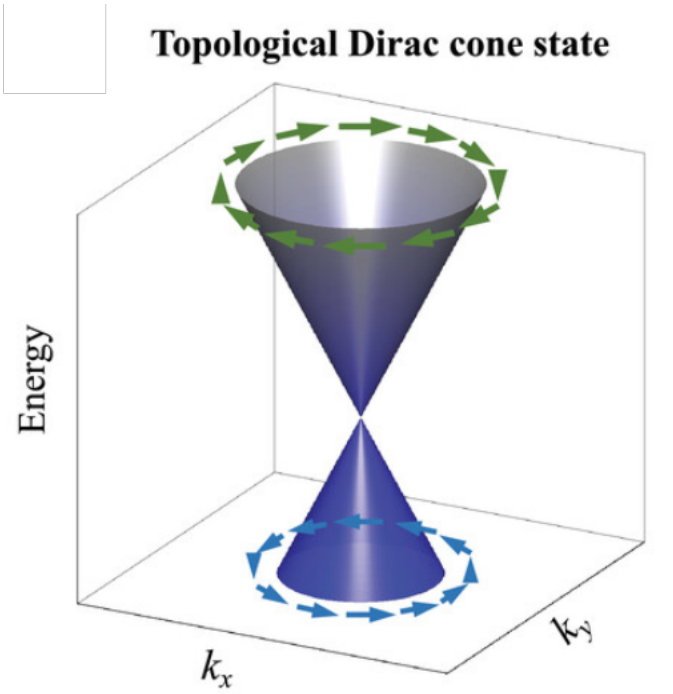

Figure 5. Dirac cone dispersion relation. Energy is plotted against momentum in the two axes of the surface. The two possible orientations of spin relative to momentum is also shown as blue and green arrows. Note that the blue orientation is always at a lower energy than the green orientation. Also note that the Fermi level is at the intersection of the cones.

We are working with the group to fabricate quantum walls on differently oriented substrates, specifically (111) instead of (100). We expect that a device fabricated on (111) orientated substrate will exhibit stronger nontrivial behavior, including a widening of the inverted band gap.

4 Comparison to Conventional Materials

Topological materials are not fully understood yet. However, for relatively well-known topological materials (especially TIs) like Bi_2Se_3, efforts are already underway to apply their unique properties to solve real-world problems.

4.1 Topological FETs

The most obvious reaction after encountering a new material system like topological insulators is to try to make a transistor out of it. Topological insulators seem especially fit in a transistor because the materials inherit backscattering protection due to spin-momentum locking suggests

that it might lend itself to desirable transistor characteristics like high electron mobility [15]. However, an immediate question is how the transistor would be switched: the metallic surface state of a TI suggests that modulating carrier density would be difficult if not impossible.

The first studies in topological field effect transistors (TIFETs) used Bi_2Se_3 as the channel material. A FET was constructed using a 100nm film of Bi_2Se_3 with gates on top and bottom to independently control carrier densities. Evidence from the fabricated device suggested that current was able to flow through three channels of the material: the top and bottom conducting surface states and the bulk material. Additionally, an applied electric field through the gates was able to change the dominant carrier type from electrons to holes, a property essential for eventually building CMOS circuits.

Subsequent experiments attempted to reduce the electron density in the bulk state. The most direct way was to shrink the device: sufficiently small samples of TI thin films exhibited the off state, indicating the depletion of bulk doping. However, miniaturization was not enough to invert the doping type. Instead, introducing Ca dopants into the Bi2Se3 crystals was necessary to change the dominant carrier from n-type to p-type.

Continued efforts in creating a competitive TIFET converged to a design similar to conventional GAAFETs. In the design, a 50nm to 150nm wide Bi_2Se_3 nanowire is coated with a dielectric and then a conductive gate. Experimental voltage-current curves showed promising signs for TI-based CMOS processes. However, two main characteristics of the device are of concern. First, the measured carrier mobility decreased significantly as the ambient temperature was increased to normal operating temperatures: at 77K the electron mobility was $1300\frac{cm^2}{Vs}$ while at room temperatures it decreased to $100\frac{cm^2}{Vs}$. This significantly underperforms nanowires made from silicon and suggests that switching behavior is primarily realized by the bulk, neglecting any advantages from surface backscattering protection. Secondly the sub-threshold swing of the constructed TIFET is poor compared to conventional transistors and especially tunneling FETs. The sub-threshold swing is directly related to the voltage required to operate the FET and as the industry moves towards lower voltages capable of faster switching and lower power consumption, the high sub-threshold swing may be another dealbreaker.

In addition to experimental studies of Bi_2Se_3 based FETs, many theorical and experimental studies of FETs based on other topological materials are underway. One study proposed a transistor that used the gate to modulate backscattering in HgTe nanoribbons. Another study used the gate to control the energy gap within the material by coupling edge states to current channels. A third study used an electric field to control whether the TI was in its topological state or its trivial state. A fourth used the piezoelectric effect to modulate the phrase of the TI as a switching mechanism. Clearly, the unique mechanics of topological insulators give way to a myriad of possibilities in creating transistors.

Figure 6 compares potential FET designs based on conventional materials and topological insulators. The established benchmarks of switching delay and switching energy are shown. For example, high-performance FETs consume the most energy, but they also have the highest switching speeds, making them competitive. On the other hand, the homojunction tunnel FETs use an order-of-magnitude less energy but has a much higher switching delay. The figure shows that TI-based FETs can potentially bring several order-of-magnitude improvements on both metrics, motivating continued research into TIFETs even if they are not yet viable.

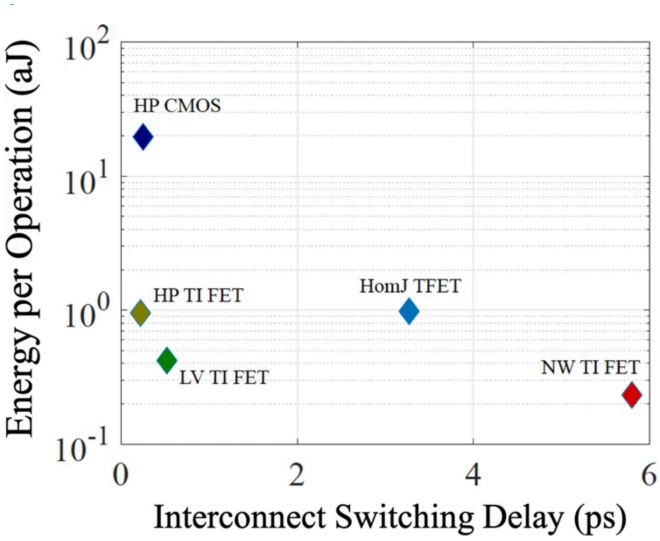

Figure 6. Comparison of next generation FET designs based on conventional materials and topological materials. Future TI-based FET designs may provide order-of-magnitude improvements in the standard industry benchmarks of switching delay and switching energy.

4.2 Magnetic Random Access Memory

The previous section focused on the exploitation of TIs innate backscattering protection. However, the ability of TIs to manipulate spin lends itself to other technologies as well.

Magnetoresistive random-access memory (MRAM) is a type of memory that stores data by changing the magnetization alignment of a ferromagnetic material [16]. The primary component in MRAM is the magnetic tunnel junction, which consists of two ferromagnetic plates stacked on top of each other with a thin insulating film in between. When a voltage applied between the two plates, electrons can tunnel through the insulating film and create a current. If the two plate are magnetized in the same direction, tunnelling is more efficient, and the current is higher. Conversely, if the plates are magnetized in opposite directions, the current is lower. This is because electrons orient their

spins to the magnetic field theyre in. For an electron to tunnel to a material with a different magnetic orientation, it would have to additionally change its spin, reducing the probability for the tunneling to occur.

Reading from a magnetic tunnel junction is as simple as measuring its resistance, but writing to it requires changing the magnetic alignment of one of the plates (referred to as the writable plate). The first MRAM devices accomplished this by inducing a magnetic field at the writable plate using a current, i.e. by creating an electromagnet. However, this design suffered from high power consumption because of the magnitude of current required to create the magnetic field. Additionally, as the device is scaled down, the field tends to overlap with adjacent cells, causing additional unwanted writes. This limits the storage densities that MRAM devices can reach.

Second generation MRAM devices use a different phenomenon to remagnetize the writable plate: spin-transfer torque (STT). When an electron changes its spin to coincide with the magnetic alignment of the surrounding material, the orientation of the material is also slightly changed. If enough electrons are injected with opposite spins, the orientation of the material can be swapped, constituting a write. Using spin-transfer torque to reorient the writable plate uses significantly less energy than inducing a magnetic field and additionally allows the cell to miniaturized beyond scales possible in first generation MRAM devices.

A separate phenomenon, spin-orbit torque (SOT) can also be used to change the magnetic orientation of a material [17]. It relies on coupling a material with strong spin-orbit coupling (SOC) next to a ferromagnetic material. When a current passes through the material with SOC, a spin polarized current is generated. This current then changes the alignment of the ferromagnetic material. Typically the current required for SOT is much lower than that required for STT.

The mechanism for SOT in topological insulators is slightly different [15]. Recall that spin-momentum locking causes spin to be locked perpendicular to the electrons momentum. Should an electric field be applied to the topological insulator, a Hall current is generated. However, because of spin-momentum locking, the spins of the electrons in the Hall current disproportionally accumulate in one direction. This creates the spin polarized current.

As it turns out, SOT in topological insulators is much stronger than SOT in other SOC materials. Refer to Figure 7 for a comparison. While metals are more conductive than TIs, certain TIs have a comparable if not much higher spin Hall conductivity. Additionally, the spin Hall angle, which to some extent represents the ability for the material to generate spin polarized currents, is much higher for TIs. Both results show that TIs are a strong candidate as building blocks for future MRAM technologies.

5 Conclusion

Topological materials are one of the most promising frontiers in material science. By mathematically characterizing materials' electronic properties, the field has illuminated exotic substances with novel properties. In this paper we discussed the mathematical and theoretical origins of the field, then covered its evolution thereon. Originating from the effort to study the quantum hall effect, topological materials now encompass several generations of materials.

One such class of materials are topological insulators, which host unique properties like spin-momentum coupling and protected surface states. Research into applying these properties for solving real-world problems is already underway, and we covered two of these potential applications: topological FETs and MRAM. However, understanding of topological materials is far from complete, and topological materials are under intense research efforts. We covered one such ongoing research effort occurring right here at Tufts.

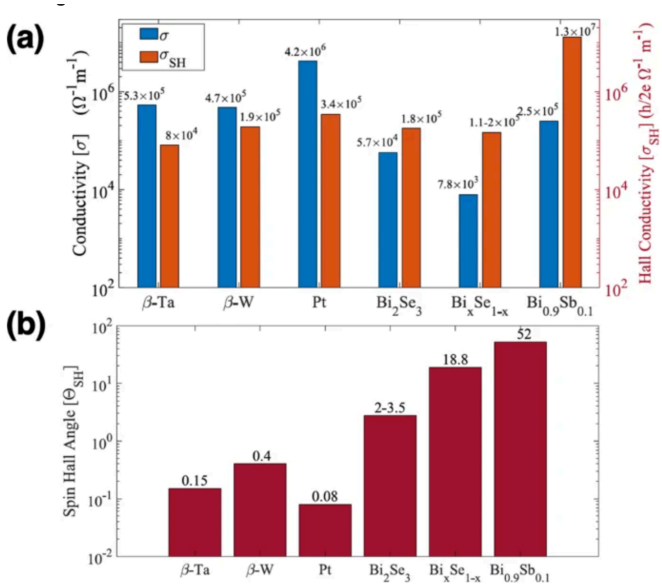

Figure 7. Comparison of conventional SOT metals and TIs. In (a), the conductivity and hall conductivity are ranked. While metals are more conductive than TIs due to a higher density of states, TIs have higher hall conductivity. In (b), the spin hall angle of different materials are compared and again shows potential for TIs.

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